The Construction of Optimal Portfolio Using Sharpe's Single Index Model - An Empirical Study on Nifty Metal Index  

– Dr. J. Murthy*

Abstract

While making investment decisions particularly in equity market, risk and return plays an important role perhaps the most relevant question is which stocks should be placed in the portfolio matters a lot. A good combination of equity stocks in the portfolio will give better return for a given level of risk.

The main focus of this research is to construct an optimal portfolio in Indian stock market with the help of the Sharpe single index model. In this study, totally 14 metal stocks have been selected from iron and steel industry and these stocks are constituent of the NSE Nifty metal index. The monthly data for all the stocks for the period of January 2012 to December 2016 have been considered. The study reveals that only two company stocks constitute the optimum portfolio and these are Vēdanta and Tata steel with an ideal proportion of investment of 86.37% and 13.62% respectively.

Keywords: Beta, Market variance, unsystematic risk, Single index model, optimal portfolio, Risk and return trade off, Diversification, Nifty.

Introduction

The security analysis and portfolio management has emerged as the most important aspect for rational investment and decision making. Portfolio is a combination of securities such as stocks, bonds and money market instruments. The process of blending together the broad assets classes so as to obtain optimum return with minimum risk is called portfolio construction. A portfolio tries to trade off the risk return preferences of an investor by not putting all eggs in single basket and thus allows for sufficient diversification. Markowitz was the first who laid foundation of "Modern portfolio theory" to quantify risk. He provided analytical tools for analysis and selection of optimal portfolio. This portfolio approach won him Nobel Prize in 1990. The work done by Markowitz was extended by William Sharpe. He simplified the amount and type of input data required to perform portfolio analysis. He made the numerous and complex computations easy which were essential to attain optimal portfolio. This simplification is achieved through single index model. This model proposed by Sharpe is the simplest and the most widely used one.

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The study focuses on finding out an optimal portfolio using Sharpe's single index model. This article is framed around constructing an optimal portfolio by balancing the positive and negative correlation existing between the securities and in turn getting returns closer to the anticipated results. For this study, the stocks from the iron and steel industry have been selected as this sector is of prime importance for real growth of an economy.

Objectives of the Study

The primary objective is to construct an optimal stock portfolio among the selected metal stocks. The specific objectives are:

- To present a review of past works relating to optimum portfolio construction and analysis.
- To build an optimum stock portfolio among selected stocks using Sharpe's Single Index model.
- To calculate the proportion of investment to be made into each of the stock that is included in the optimal stock portfolio that is constructed using Sharpe index model.

Review of Literature

Dr. Sathya Swaroop Debasish, Jakki Samir Khan (2012), in their research found that only three company stocks constitute the optimum portfolio and these are Asian Paints, Tata motors and Hero Motor Corp. with ideal proportion of investment of 1.9 %, 38.88 and 58.22% respectively.

Saravanan and Natarajan (2012) attempted to construct an optimal portfolio by using Sharpe's Single Index Model. For this purpose NSE Nifty Index has been considered. The daily data for all the stocks and index for the period of April 2006 to December 2011 have been considered. They formulated the cut-off point and selected stocks having excess of their expected return over risk free rate of return surpassing this cut-off point. Percentage of investment in each of selected stocks is then decided on the basis of respective weights assigned to each stock depending on respective beta value, stock movement variance unsystematic risk, return on stock and risk free return vis-à-vis the cut off rate of return. From the empirical analysis, it was concluded that returns on either individual securities or on portfolio comprises of securities of different companies listed in Nifty 50 stocks under various sectors are asymmetrical and heterogeneous. The optimal portfolio consists of four stocks selected out of 50 short listed scrips, giving the return of 0.116. Further it helps to elicit that return on securities of different portfolio is independent of the systematic risk prevailing in the market.

In the Indian scenario, Varadharan (2011) constructed an optimal equity portfolio with the help of Sharpe Index model. The study was conducted with the financial data from April 2006 to March 2011. The sample size was limited to 19. He took these companies from Banking and Information Technology. The portfolio was constructed with the top 5 stocks that meet the criteria to be included in the portfolio according to Sharpe Index Model. The portfolio predominantly consisted of stocks from the banking sector, and one stock from IT sector.
Sandip Mukherji (2010), identified the optimal portfolios for three different objectives relating to risk and return for three different holding periods of 1, 5 and 10 years. Their study only considers portfolios consisting treasury bills and stocks only. Bootstrap method was used to generate returns for holding periods and to analysis of their composition was done. Risk-Return analysis of the portfolios examined shows higher returns in case of riskier returns. Results of this study shows that optimal portfolios contain increasing allocations of risky assets and decreasing allocations of safe assets as the holding period increases.

AJ Du Plessis, M Ward (2009), explained the possibility of identifying the right kind of attributes in a portfolio ensuring good returns in the future. It stresses on the importance of applying the Markowitz model of portfolio selection with respect to the Johannesburg Securities Exchange to analyse if an optimal portfolio can be identified and be used as an effective guideline method for trading purposes. It concludes on how an optimal portfolio is selected and rebalanced periodically and comparing the returns with the JSE index rate thereby, showing how a particular trading strategy can outperform in the market.

Frank J Fabozzi, Harry M Markowitz, Francis Gupta (2008) stated that the main objective of portfolio selection is the grouping and construction of the portfolios to maximize returns expected on them with a certain level of risk. Investors generally use modelling techniques on the historical data and based on their future expected returns, the optimal portfolio selection is done and this allows computing the investment risk and expected return on a portfolio. Also, the theory of portfolio selection shifts from individual risk to the entire risk of the portfolio and shows the possibility of combining risky assets to produce an optimum portfolio.

Rainer Baule (2008) analysed optimal portfolio selection for small investors considering transaction costs and risk costs. Portfolio theory cannot be directly applied to small investors due to limitations in form of transaction and risk costs. An empirical study done ascertains that for smaller volume of investment, transaction costs dominate risk costs so that optimal portfolio contains only very small number of assets. Based upon these findings, a comparative study between direct investments in stock or in index certificates and ETF’s is done. The findings of this study show that direct investment is most advantageous than alternative investment products if sum of transaction costs and risk costs is smaller than annual costs.

Research Methodology

This is a descriptive study on the optimal portfolio construction of stocks. The data taken for the research is secondary in nature. The data has been collected from the official website of National Stock Exchange (NSE), namely www.nseindia.com. The study is conducted with the financial data for the past 4 years from January 2012 to December 2016. The sample size of the study is limited to 14 metal stocks and monthly opening, closing stock prices have been taken and these selected stocks are also constituents of Nifty metal index. The sampling technique adopted is purposive sampling.
Here 0.565 percent is considered as risk free rate for one month based on the portfolio on 364 days Government of India treasury bills.

**Return**

The monthly return on each of the selected stocks is calculated with the following formula.

\[
\text{Stock return} = \frac{P_1 - P_0}{P_0}
\]

Where \(P_0\) = Initial stock price

\(P_1\) = Ending stock price (period 1)

Risk; According to Sharpe's single index model risk is divided into two that is systematic risk and unsystematic risk. Systematic risk is \(\sigma^2x \beta^2\) and unsystematic risk is \(\sigma^2_{ei}\).

**Steps in Construction of Optimal Portfolio Using Single Index Model**

This model firstly ranks the securities based on their excess return to beta ratio. After that all securities are arranged according to their ranks. Then cutoff rate is calculated and it is compared with excess return to beta for deciding whether to select the security for investment or not. The model explains the weight that should be allocated to each security to obtain optimal portfolio.

**Step 1:** Calculate excess return to beta ratio for each security under consideration

\[
\text{Excess return to beta ratio} = \frac{(R_i - R_f)}{\beta_i}
\]

Where

\(R_i\) = Expected return of Security i

\(R_f\) = Risk free rate of return Present MIBOR rate is taken as risk free rate Rf

\(\beta_i\) = the Beta co-efficient of the security or excess return of the security over market index

**Step 2:** Rank the securities based on the excess return to beta ratio.

**Step 3:** Calculate the cut of rate using the formulae. Highest cut off rate will be regarded as \(C^*\)

Where

\(\sigma_m^2\) = Market variance

\(R_i - R_f\) = Market risk premium

\(\sigma_{ei}^2\) = Unsystematic risk of the security

\(\beta_i\) = systematic risk of the security

**Step 4:** Selection of securities for investment. If \(\frac{(R_i - R_f)}{\sigma_i}\) is greater than cut off rate then the security will be included in the portfolio.
Step 5: Calculate the proportion to be invested in each security is calculated.

\[ X_l = \frac{z_i}{\sum_{i=1}^{N} z_i} \]

Where

\[ z_i = \frac{\beta_i}{\sigma_i^2} \left( \frac{R_i - R_f}{\beta_i} - C^* \right) \]

C* is the cut off rate

According to the Sharpe model the excess return of any stock is directly related to its excess return to beta ratio. It measures the additional return on a security (excess of the risk less asset return) per unit of systematic risk. The ratio provides a relationship between potential risk and reward. For the calculation of this ratio, the risk-free return (Rf) is taken as the rate of return on the 364-days is 6.78 and the monthly average of Treasury bill which is found to be 0.565% for the period under study.

The study has been done in two stages

**Stage 1: calculation of the Cutoff point**

The model helps us in finding the cutoff point (Cut off rate of return) and selecting stocks having excess return to beta ratio and surpassing this cut-off point.

**Stage 2: Construction of optimal portfolio**

In the second stage Percentage of investment in each of selected stocks is then decided on the basis of respective weights assigned to each stock depending on respective beta value, stock movement variance unsystematic risk, return on stock and risk free return vis-a-vis the cut off rate of return.

**Data Analysis and Interpretation:**

This part of the study deals with data analysis and interpretation done for 14 companies (listed in the Nifty metal index). From this table, it may be observed that each company is assigned a rank on the basis of its excess return to beta ratio.
It can be seen from the table that Vedanta yielded the maximum excess return to beta ratio (13.68) and ranked 1 among the companies selected followed by Tata steel with (4.641) and JSW yielded lowest excess return to beta of - 20.24.

Table 2: Showing cut off point of sample companies

<table>
<thead>
<tr>
<th>Company</th>
<th>Ri-Rf/βi</th>
<th>(Ri-Rf) * βi/ σ²</th>
<th>(Ri-Rf) βi/σei²</th>
<th>βi2/σei²</th>
<th>? βi2/σei²</th>
<th>ci</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vedanta</td>
<td>13.69</td>
<td>0.22</td>
<td>0.22</td>
<td>0.02</td>
<td>0.02</td>
<td>3.00</td>
</tr>
<tr>
<td>Tata steel</td>
<td>4.64</td>
<td>0.77</td>
<td>1.00</td>
<td>0.17</td>
<td>0.18</td>
<td>4.13</td>
</tr>
<tr>
<td>Hindalco</td>
<td>1.87</td>
<td>0.38</td>
<td>1.37</td>
<td>0.20</td>
<td>0.38</td>
<td>3.10</td>
</tr>
<tr>
<td>APL Apollo</td>
<td>1.84</td>
<td>0.35</td>
<td>1.72</td>
<td>0.19</td>
<td>0.57</td>
<td>2.72</td>
</tr>
<tr>
<td>Hind zinc</td>
<td>1.18</td>
<td>0.08</td>
<td>1.80</td>
<td>0.07</td>
<td>0.64</td>
<td>2.57</td>
</tr>
<tr>
<td>Welspan corp</td>
<td>0.15</td>
<td>0.02</td>
<td>1.82</td>
<td>0.14</td>
<td>0.78</td>
<td>2.17</td>
</tr>
<tr>
<td>Coal India</td>
<td>-0.46</td>
<td>-0.07</td>
<td>1.75</td>
<td>0.14</td>
<td>0.92</td>
<td>1.79</td>
</tr>
<tr>
<td>NMDC</td>
<td>-1.54</td>
<td>-0.08</td>
<td>1.67</td>
<td>0.05</td>
<td>0.97</td>
<td>1.62</td>
</tr>
<tr>
<td>Orissa minerals</td>
<td>-1.63</td>
<td>-0.05</td>
<td>1.62</td>
<td>0.03</td>
<td>1.00</td>
<td>1.53</td>
</tr>
<tr>
<td>NALCO</td>
<td>-4.27</td>
<td>-0.03</td>
<td>1.60</td>
<td>0.01</td>
<td>1.01</td>
<td>1.49</td>
</tr>
<tr>
<td>Jindal steel</td>
<td>-8.56</td>
<td>-0.09</td>
<td>1.51</td>
<td>0.01</td>
<td>1.02</td>
<td>1.40</td>
</tr>
<tr>
<td>Bhushan</td>
<td>-8.75</td>
<td>-0.07</td>
<td>1.43</td>
<td>0.01</td>
<td>1.03</td>
<td>1.32</td>
</tr>
<tr>
<td>SAIL</td>
<td>-11.73</td>
<td>-0.01</td>
<td>1.43</td>
<td>0.00</td>
<td>1.03</td>
<td>1.31</td>
</tr>
<tr>
<td>JSW steel</td>
<td>-20.24</td>
<td>0.00</td>
<td>1.42</td>
<td>0.00</td>
<td>1.03</td>
<td>1.31</td>
</tr>
</tbody>
</table>

Source: computed from the stock price available at http://www.nseindia.com
Interpretation

The selection of the stocks depends on a unique cut-off rate such that all stocks with higher ratios of excess return to beta are included and stocks with lower ratio are left out. The cumulated values of Ci start declining after a particular Ci and that point is taken as the cut-off point and that stock ratio is the Cut-off Ratio C. The highest value of Ci is taken as the cut-off point that is C*. From table it is seen that Tata Steel has the highest the cut-off rate of C* = 4.13 All the stocks having Ci greater than C* can be included in the portfolio.

Calculation of Optimal Portfolio:

The proportion to be invested in each security (weight) is calculated using the following equation.

\[ X_i = \frac{z_i}{\sum_{i=1}^{N} z_i} \]

Where

\[ z_i = \frac{\beta_i}{\sigma_{\epsilon_i}} \left( \frac{R_i - R_f}{\beta_i - C^*} \right) \]

C* is the cut off rate.

The calculation of optimal portfolio is shown in the table below.

<table>
<thead>
<tr>
<th>company</th>
<th>( \beta_i )</th>
<th>( \sigma_{\epsilon_i} )</th>
<th>Ri-Rf/( \beta_i )</th>
<th>C*</th>
<th>Zi</th>
<th>Zi</th>
<th>Xi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vedanta</td>
<td>0.3868</td>
<td>9.2124</td>
<td>13.68718</td>
<td>4.13455</td>
<td>0.401089</td>
<td>0.401089</td>
<td>0.863753</td>
</tr>
<tr>
<td>Tata steel</td>
<td>1.334002</td>
<td>10.6869</td>
<td>4.641297</td>
<td>4.13455</td>
<td>0.063267</td>
<td>0.464356</td>
<td>0.136247</td>
</tr>
</tbody>
</table>

Source: computed from the stock price available at http://www.nseindia.com

Interpretation

After determining the securities to be included in the optimal portfolio, we have to determine the proportion of investment in each of these stocks. Only those stocks with Excess return to beta ratio more than C* are to be selected in the optimal portfolio. It can be observed from above table that only two stocks qualify to be included in the optimal portfolio on this criterion. By using Sharpe index model, the maximum investment should be made in Vedanta with a proportion of 86.37%, and in Tata Steel with investment proportion of 13.62%. Evidently, the companies chosen for the investments are growing at a steady rate in the recent years.
Findings

Risk and return play an important role in making any investment decisions. This study aims at analyzing the opportunities that are available for investors as per as returns are concerned and the investment of risk thereof.

• Out of 14 companies taken for the study, 7 companies are showing negative return and the other 07 companies are showing positive returns.

• With regard to beta values, out of 14 companies selected, Tata steel, we spun corp and Vedanta stocks found to be aggressive (showed beta above 1, indicating that the investments in this stock is outperforming than the broader market.

• The 14 stocks that are included in NSE Nifty metal index, only two stocks namely Vedanta and Tata steel are included in the Optimal Portfolio constructed in this study with maximum suggested investment of 86 percent in Vedanta and 14 percent in Tata steel.

Suggestions

• The proportion of investment in each of the stock may change time to time hence the constructed optimal portfolio is subject to change.

• The beta and variance of the stocks may change frequently. So, Investors are suggested to observe the market continuously.

• The stocks in the portfolio must be evaluated continuously and make revisions periodically.

Conclusion

The study is conducted to test utility of Sharpe's single index model in optimal portfolio construction. From the study, it is concluded that, the 14 stocks that are included in NSE Nifty metal index, only two stocks namely Vedanta and Tata steel are included in the Optimal Portfolio constructed in this study with maximum suggested investment of 86 percent in Vedanta and 14 percent in Tata steel.

Limitations of the Study

• The study is confined to the stocks included in the Nifty Metal index.

• The result of the study may not hold good for the longer period as the stock market is so volatile in nature.
References


Websites